Trigonometry - the study of the relationships between the sides and angles of right triangles.

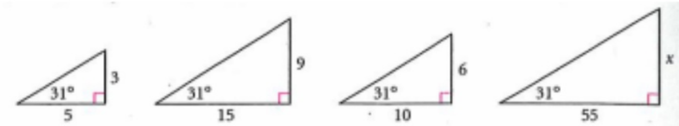
In this unit we will
- discover the sine, cosine, and tangent ratios
- discover and apply the Law of Sines
- investigate the Pythagorean Identity
- learn and apply the Law of Cosines, and
- use trigonometry to solve applied problems.

Trigonometric Ratios

4/19/18

IWBAT learn about the sine, cosine, and tangent ratios of trigonometry.

Trigonometric Ratios



What is a good approximation for *x*?

12? ~~17?~~

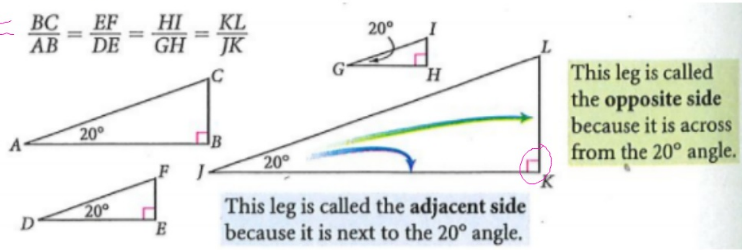
$\frac{3}{5}$ $\frac{9}{15}$ $\frac{6}{10}$ $\frac{x}{55}$

x is a factor of 3

33✓

These triangles are similar via the AA Similarity Conjecture.

$\tan 20° = \frac{BC}{AB} = \frac{EF}{DE} = \frac{HI}{GH} = \frac{KL}{JK}$



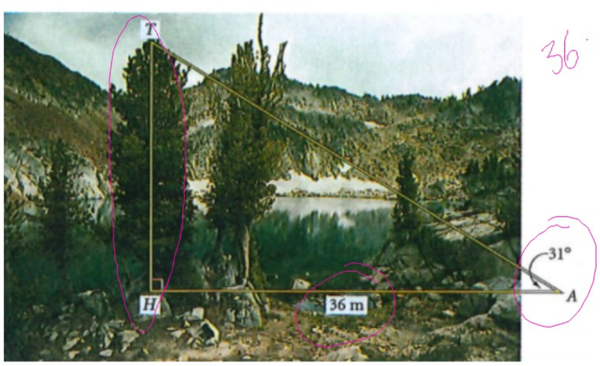
The ratio of the opposite side length to the adjacent side length is the *tangent* of the angle.

$\text{tangent } 20° = \frac{KL}{JK}$

IWBAT learn about the sine, cosine, and tangent ratios of trigonometry.

Trigonometric Ratios

At a distance of 36 m from a tree, the angle from the ground to the top of the tree is 31 deg. How tall is the tree?



O a

$36 \cdot \tan 31 = \frac{TH}{36m} \cdot 36$

$36 \tan(31) = TH$

$21.6m \sim TH$

IWBAT learn about the sine, cosine, and tangent ratios of trigonometry.

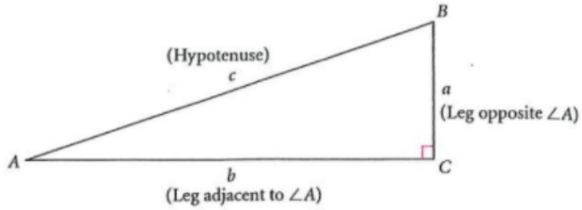
Trigonometric Ratios

Sine is the ratio of the opposite side length to the hypotenuse length.
Cosine is the ratio of the adjacent side length to the hypotenuse length.

Sine is abbreviated *sin*, cosine is abbreviated *cos*, and tangent is abbreviated *tan*.

Trigonometric Ratios

SOHCAHTOA



$\text{Sin} = \frac{O}{H}$

$\text{Cos} = \frac{A}{H}$

$\text{Tan} = \frac{O}{A}$

For an acute angle A in any right triangle ABC:

$$\text{sine of } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} \quad \text{or} \quad \sin A = \frac{a}{c}$$

$$\text{cosine of } \angle A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} \quad \text{or} \quad \cos A = \frac{b}{c}$$

$$\text{tangent of } \angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} \quad \text{or} \quad \tan A = \frac{a}{b}$$

IWBAT learn about the sine, cosine, and tangent ratios of trigonometry.

Trigonometric Ratios

Using a calculator, calculate the three trigonometric ratios for each angle. X.XXX

sin(40)=

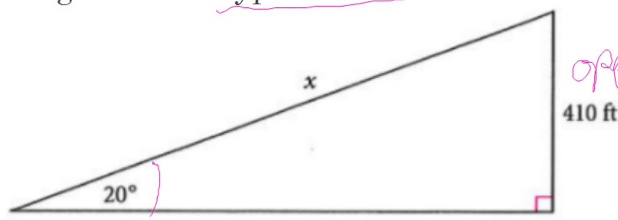
Angle measure	Sin	Cos	Tan
0°	0	1	0
10°	0.174	0.985	0.176
20°	0.342	0.940	0.364
30°	0.5	0.866	0.577
40°	0.643	0.766	0.839
50°	0.766	0.643	1.192
60°	0.866	0.5	1.742
70°	0.940	0.342	2.747
80°	0.985	0.174	5.671
90°	1	0	?

Trigonometric Ratios

Find the length of the hypotenuse.

SOHCAHTOA

SOH



$$x \sin(20) = \frac{410}{\cancel{x}}\cancel{x}$$

$$\frac{x \cancel{\sin(20)}}{\cancel{\sin(20)}} = \frac{410}{\sin(20)}$$

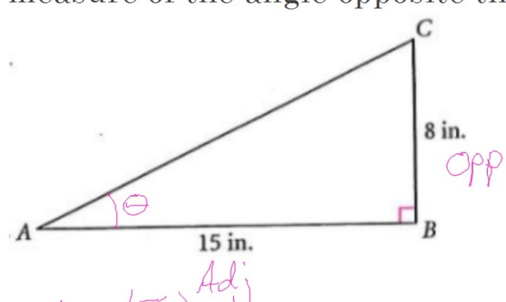
$$x = \frac{410}{\sin(20)}$$

$$x \approx 140.2 \text{ ft}$$

Trigonometric Ratios

Find the measure of the angle opposite the 8-inch leg.

SOHCAHTOA



$$\tan^{-1}(\tan(A)) = \tan^{-1}\left(\frac{8}{15}\right)$$

$$A = \tan^{-1}\left(\frac{8}{15}\right)$$

$$A \approx 28.07°$$

$$A \approx 28°$$

$$A \approx 28.1°$$

Trigonometric Ratios

Pp. 624-625 #1-7, 10, 12, 14-16, 21

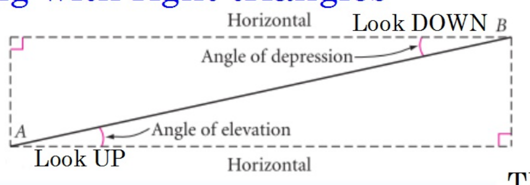
Problem solving with right triangles

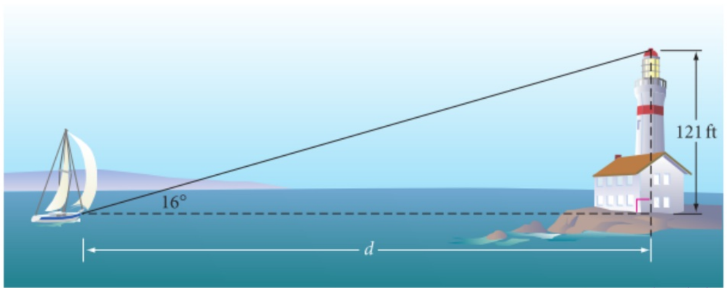
4/24/18

IWBAT use trigonometry to solve applied problems.

Problem solving with right triangles

SOHCAHTOA





The angle of elevation from a sailboat to the top of a 121 ft lighthouse on the shore measures 16°. To the nearest foot, how far is the sailboat from shore?

find the adjacent side (d)
Know angle + Opposite side

$$\frac{d \cdot \tan(16)}{\tan(16)} = \frac{121\text{ ft}}{\cancel{d}} \cdot \cancel{d} \div \tan(16)$$

$$d = \frac{121\text{ ft}}{\tan(16)} = \underline{422\text{ ft}}$$

IWBAT use trigonometry to solve applied problems.

Problem solving with right triangles

Pp. 628-629 #1-7, 9, 11, 14-16

IWBAT use trigonometry to solve applied problems.

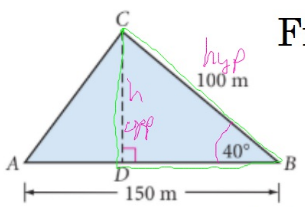
The Law of Sines

4/25/18

IWBAT discover and apply the Law of Sines.

The Law of Sines

A property called the Law of Sines allows you to find two side lengths of a triangle if you know one side length and two angle measures. It is related to the area of a triangle.



Find the area of triangle ABC.

$$A = \frac{b \cdot h}{2} = \frac{150_m \cdot h}{2}$$

SOHCAHTOA $100_m \cdot \sin(40°) = \frac{h}{100_m} \cdot 100_m$

$$100_m \sin(40) = h$$

$$A = \frac{150_m \cdot 100_m \sin(40)}{2}$$

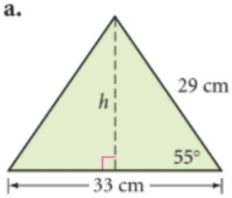
$$A = \sim 4821 m^2$$

IWBAT discover and apply the Law of Sines.

The Law of Sines

Find the area of each triangle.

a.



$A = \frac{33 \cdot h}{2}$

$29 \cdot \sin(55) = \frac{h}{29} \cdot 29$

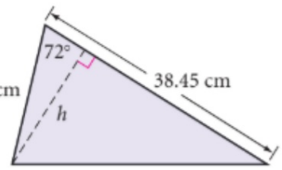
$29 \cdot \sin(55) = h$

$A = \frac{33 \cdot 29 \cdot \sin(55)}{2}$

$A = 392 \text{ cm}^2$

$A = \frac{b \cdot a \cdot \sin(c)}{2}$

b.



$A = \frac{38.45\text{cm} \cdot h}{2}$

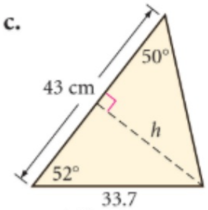
$21\text{cm} \cdot \sin(72) = \frac{h}{21\text{cm}} \cdot 21\text{cm}$

$21\text{cm} \sin(72) = h$

$A = \frac{38.45\text{ cm} \cdot 21\text{cm} \sin(72)}{2}$

$A = 383.9 \text{ cm}^2$

c.



$A = \frac{43 \cdot h}{2}$

$33.7 \cdot \sin(52) = \frac{h}{33.7} \cdot 33.7$

$33.7 \cdot \sin(52) = h$

$A = \frac{43 \cdot 33.7 \cdot \sin(52)}{2}$

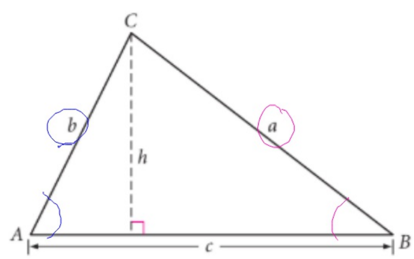
$A = 571 \text{cm}^2$

IWBAT discover and apply the Law of Sines.

The Law of Sines

SAS Triangle Area Conjecture

The area of a triangle is given by $A =$ $a \cdot b \cdot \sin(C)/2$, where a and b are the lengths of two sides and C is the angle between them.



- Find h in terms of a and the sine of an angle.
- Find h in terms of b and the sine of an angle.
- Show $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$

$h = a \cdot \sin(B)$

$h = b \cdot \sin(A)$

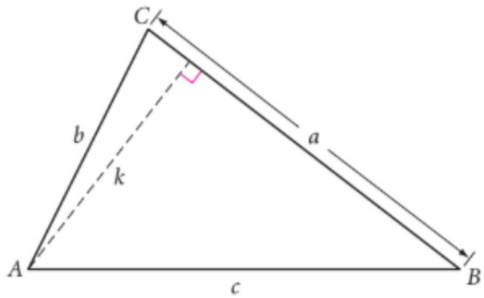
$\frac{\frac{\cancel{a}\sin(B)}{b}}{\cancel{a}} = \frac{\frac{\cancel{b}\sin(A)}{\cancel{b}}}{a}$

$\frac{\sin(B)}{b} = \frac{\sin(A)}{a}$

IWBAT discover and apply the Law of Sines.

The Law of Sines

For this same triangle and height k:



- Find k in terms of c and the sine of an angle.
- Find k in terms of b and the sine of an angle.
- Show $\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

$k = c\sin(B)$

$k = b\sin(C)$

$\frac{\cancel{c}\sin(B)}{b} = \frac{\cancel{b}\sin(C)}{\cancel{b}}$

$\div \cancel{c} \qquad \div c$

$\frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

IWBAT discover and apply the Law of Sines.

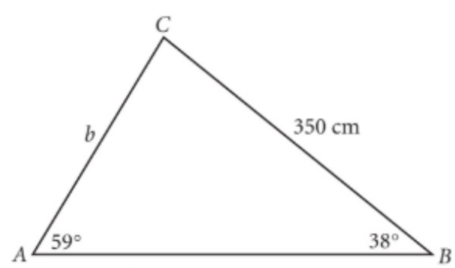
The Law of Sines

Law of Sines

For a triangle with angles A, B, and C, and side lengths of a, b, and c (a opposite A, b opposite B, and c opposite C),

$\frac{\sin(A)}{?\,a} = \frac{?\,\sin(B)}{b} = \frac{?\,\sin(C)}{?\,c}$

Given triangle ABC, find b.



380 $\quad \frac{\sin(38)}{b} = \frac{\sin(59)}{350\text{cm}}$

$b \cdot \quad \frac{350\text{cm}\sin(38)}{b} = \sin(59)$

$\div\sin(59) \quad 350\text{cm}\sin(38) = b\sin(59)$

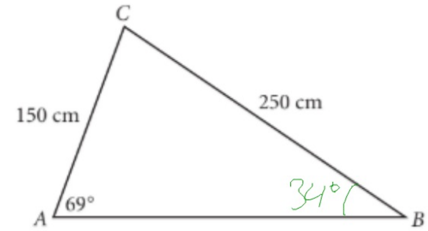
$\frac{350\sin(38)}{\sin(59)}\text{cm} = b$

$b \sim 251.4\text{ cm}$

IWBAT discover and apply the Law of Sines.

The Law of Sines

Find the measure of angle B.



$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$150\text{cm} \cdot \frac{\sin(69)}{250\text{cm}} = \frac{\sin(B)}{\cancel{150\text{cm}}} \cdot \cancel{150\text{cm}}$$

$$\sin^{-1}\left(\frac{150\cancel{\text{cm}}\sin(69)}{250\cancel{\text{cm}}}\right) = \sin^{-1}(\sin(B))$$

$$34° = B$$

IWBAT discover and apply the Law of Sines.

The Law of Sines

Pp. 637-638 #1, 2, 5-11

IWBAT discover and apply the Law of Sines.

The Law of Cosines

IWBAT investigate the Pythagorean Identity and learn & apply the Law of Cosines.

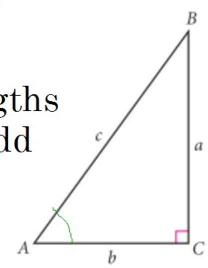
The Law of Cosines

You can derive trigonometric relationships from the Pythagorean Thoerem. Choose a value for angle A. Find $\sin^2(A) + \cos^2(A)$. Repeat for several values of A. Make a conjecture based on your results.

$\sin^2(8)+\cos^2(8)=1$
$\sin^2(1080)+\cos^2(1080)=1$
$\sin^2(10)+\cos^2(10)=1$
$\sin^2(999)+\cos^2(999)=1$

$\sin^2(A)+\cos^2(A)=1$ for all values of A

Using the triangle at right, substitute side lengths into the right-hand side of this equation and add the fractions.



$$\sin^2(A) + \cos^2(A) = \left(\frac{?}{?}\right)^2 + \left(\frac{?}{?}\right)^2$$

$$1 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2}{c^2} + \frac{b^2}{c^2}$$

SOHCAHTOA

$$c^2 \cdot 1 = \frac{a^2+b^2}{\cancel{c^2}} \cdot \cancel{c^2}$$

$$c^2 = a^2+b^2$$

The Law of Cosines

Pythagorean Identity

For any angle A, $\sin^2(A) + \cos^2(A) = 1$.

If angle $C < 90$, $c^2 = a^2 + b^2$ - something.
If angle $C > 90$, $c^2 = a^2 + b^2$ + something.
That something is $2ab*\cos(C)$.

Law of Cosines

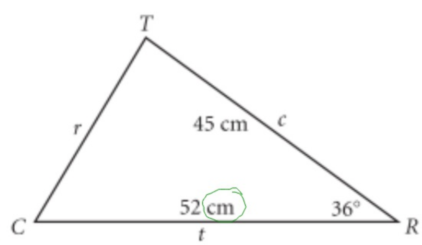
For a triangle with angles A, B, and C, and side lengths of a, b, and c (a opposite A, b opposite B, and c opposite C),
$c^2 = a^2 + b^2 - 2ab*\cos(C)$.

You can use the Law of Cosines when you know all three sides (SSS) or two sides and the included angle (SAS) of a triangle.

IWBAT investigate the Pythagorean Identity and learn & apply the Law of Cosines.

The Law of Cosines

Find r in triangle CRT.



$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$

R

$r^2 = (45)^2 + (52)^2 - 2(45)(52)\cos(36)$

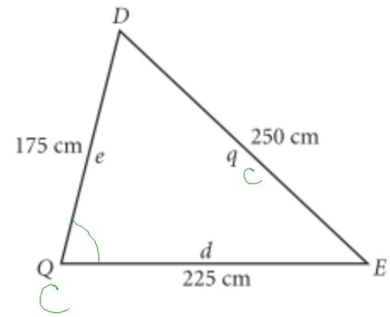
$\sqrt{r^2} \approx \sqrt{942.8005}$

$r \approx 30.70$ cm

IWBAT investigate the Pythagorean Identity and learn & apply the Law of Cosines.

The Law of Cosines

Find the measure of angle Q in triangle QED.



$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

$$250^2 = 175^2 + 225^2 - 2(175)(225)\cos(Q)$$

$$\frac{250^2 - 175^2 - 225^2}{-2(175)(225)} = \frac{\cancel{-2(175)(225)}\cos(Q)}{\cancel{-2(175)(225)}}$$

$$\left(\frac{250^2 - 175^2 - 225^2}{-2(175)(225)}\right) = \cos(Q)$$

$\cos^{-1}$ $\cos^{-1}$

$$Q = \cos^{-1}\left(\frac{(250^2 - 175^2 - 225^2)}{(-2(175)(225))}\right) = \cos^{-1}((250^2-175^2-225^2)/(-2\cdot 175\cdot 225))$$

$$C = \cos^{-1}((c^2-a^2-b^2)/(-2ab))$$

$$Q \approx 76.23^\circ$$

IWBAT investigate the Pythagorean Identity and learn & apply the Law of Cosines.

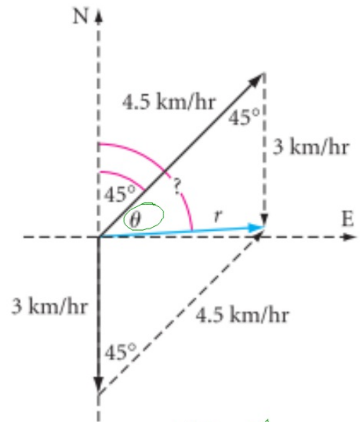
The Law of Cosines

Pp. 643-644 #1-6, 8, 11

IWBAT investigate the Pythagorean Identity and learn & apply the Law of Cosines.

Problem solving with trigonometry

Rowing instructor Calista is in a stream flowing north to south at 3 km/h. She is rowing northeast at a rate of 4.5 km/h. At what speed is she moving? In what direction is she actually moving?



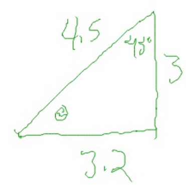
$r^2 = 3^2 + 4.5^2 - 2(4.5)(3)\cos(45)$

$r^2 = 10.15 \quad r = 3.2$ km/h

$\sin(\theta) = \frac{3}{4.5} \quad \sin^{-1}\left(\frac{3}{4.5}\right) = \theta \quad \theta = 42°$

heading 87°

$\frac{\sin(\theta)}{3} = \frac{\sin(45)}{3.2} \quad \theta = 42°$



L. cos

$c^2 = a^2 + b^2 - 2ab\cos(C)$

L. seno

$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$

IWBAT solve applied problems using trigonometry.

Problem solving with trigonometry

Pp. 648-649 #2-4, 6-8

IWBAT solve applied problems using trigonometry.

Chapter 12 Review

Pp. 659-660 #1, 2, 3, 5, 8, 13, 15, 16, 17